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Problem-Based Learning and Its Impact on 21st Century Competencies for the Study of Quadratic Functions

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Abstract: One of the most significant challenges in mathematics education stems from the mismatch between the competencies demanded by contemporary society and the conventional teaching approaches still widely employed. This disparity is particularly evident within the Ecuadorian education system. Accordingly, this study examined the influence of Problem-Based Learning (PBL) on the development of 21st century competencies during the teaching of quadratic functions among first-year high school science students. Guided by a positivist paradigm, the research adopted a quasi-experimental design involving 56 participants, equally divided into a control group (N=28), which received traditional instruction, and an experimental group (N=28), which was taught using PBL. A five-stage instructional sequence, designated IDRAC and grounded in constructivist learning theory, was developed to support the intervention. Data were collected using three instruments: a pre-test/post-test achievement assessment validated through a pilot study (Pearson's $r = 0.696$), an instructional sequence implementation checklist demonstrating acceptable internal consistency (Cronbach's $\alpha = 0.791$), and the TAC (Attitudes) Questionnaire v5.11. Baseline analysis confirmed that the two groups were statistically comparable before the intervention, as indicated by the Mann-Whitney U test ($p = 0.238$). Following the intervention, the experimental group demonstrated a statistically significant improvement in achievement between the pre-test and post-test ($p = 0.018$). The greatest gains were observed in problem-solving ability (MO +21.43 percentage points) and conceptual understanding (MO +16.07 percentage points). Furthermore, 82.14% of students in the experimental group reported favourable attitudes towards the PBL approach. Overall, the findings suggest that PBL effectively promotes the acquisition of 21st century competencies, although variations across competency domains highlight the need for stronger instructional scaffolding to enhance mathematical reasoning, communication, self-assessment, and peer-assessment practices.

Keywords: Problem-Based Learning, 21st Century Competencies, Mathematics Education, Quadratic Functions, Quasi-Experimental Design.

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1. Introduction

Within many junior and senior high school mathematics classrooms, a familiar pattern can be observed. Students accurately follow the procedural steps required to determine the roots of a quadratic equation before recording the solution in their notebooks. Yet, when asked to explain the practical significance of the discriminant or interpret the vertex of a parabola within a real-life context, many struggle to provide an answer. This recurring classroom experience, observed by the researchers over several years of teaching, reflects a broader disconnect between current instructional practices, the intended learning outcomes, and the competencies expected of learners in contemporary society.

Modern education requires students to develop competencies that extend far beyond procedural knowledge. According to Voogt and Roblin (2012), the competencies associated with the 21st century primarily comprise higher-order thinking skills that enable individuals to function effectively in increasingly complex, interconnected, and rapidly evolving environments. These competencies commonly include critical thinking, problem solving, creativity, communication, collaboration, digital literacy, self-direction, independent learning, and personal responsibility across educational settings and cultural contexts (Castellanos Monroy & Rojas Villamil, 2023). Rather than representing supplementary curricular elements, these capabilities have become fundamental outcomes of education and are essential for participation in today's workforce.

The importance of these competencies is particularly evident in mathematics education. The acquisition of mathematical reasoning is closely linked with several neuropsychological processes, including spatial ability, attention, memory, inhibitory control, and executive functioning. Within the Ecuadorian secondary curriculum, the study of quadratic functions requires learners to demonstrate proficiency in conceptual understanding, mathematical modelling, problem solving, mathematical communication, reasoning, and argumentation. When these mathematical competencies are integrated with broader 21st century capabilities, students become better equipped to apply mathematical concepts purposefully and effectively when addressing authentic problems encountered in everyday life.

Despite these curricular expectations, evidence indicates that classroom practice frequently falls short of achieving this integration. Donoso Osorio et al. (2020) argued that mathematics instruction often prioritises content delivery instead of encouraging students to investigate how mathematical ideas can be explored and constructed. Consequently, learning frequently becomes procedural, repetitive, and centred on memorising solution methods. Although students are regularly presented with contextualised exercises, many of these activities merely disguise routine procedural tasks rather than posing authentic mathematical problems. Unlike algorithmic exercises, genuine problems require learners to analyse unfamiliar situations, formulate solution strategies, and synthesise multiple mathematical concepts before reaching a conclusion.

PBL offers a pedagogical alternative capable of addressing these shortcomings. Prior literature described PBL as a learner-centred instructional strategy in which knowledge is acquired through the resolution of authentic problems. Within this approach, emphasis shifts away from simply obtaining the correct solution and instead focuses on identifying what information is required, determining how it can be obtained, deciding how it should be applied, and evaluating the effectiveness of the proposed solution. Such learning processes naturally cultivate many of the competencies regarded as essential for the 21st century. A growing body of international evidence supports the educational value of PBL in mathematics. Laine and Mahmud (2022), through a systematic literature review, concluded that PBL consistently improves academic achievement, learner motivation, and critical thinking. Similarly, Salama et al. (2025) reported substantial improvements in students' mathematical communication following the implementation of active learning methodologies. Silitonga et al. (2025) further demonstrated that PBL contributes not only to cognitive development but also to affective outcomes by strengthening learners' interest in mathematics and fostering more positive attitudes towards the subject.

Comparable findings have also emerged from research conducted in Ecuador. Buri Sánchez (2024) found that PBL significantly enhanced academic performance in learning systems of linear inequalities. Gordón Parra (2022) documented favourable educational outcomes among sixth-grade Basic General Education students following its implementation. Scholars also identified positive effects of PBL on the development of logical-mathematical thinking. Prior studies also demonstrated that the superior performance observed in the experimental groups resulted from the instructional intervention itself rather than differences between participant groups.

Although these findings provide encouraging evidence, important gaps remain in the existing literature, both of which are addressed in the present study. First, previous investigations have generally concentrated on overall academic achievement while paying limited attention to the individual mathematical competencies influenced by PBL, including conceptual understanding, mathematical modelling, problem solving, communication, reasoning, and argumentation. Consequently, it remains unclear which competencies benefit most from this instructional approach and which require additional pedagogical support or scaffolding. Second, numerous earlier studies relied primarily on descriptive statistical analyses without employing inferential procedures capable of determining whether observed improvements were statistically significant. As noted by Bautista-Díaz et al. (2020), the absence of inferential statistical analysis prevents researchers from establishing whether observed differences reflect genuine intervention effects or merely occurred by chance.

To address these research gaps, the present study sought to answer two principal questions. First, does PBL produce different levels of development across specific mathematical competencies among first-year science students studying quadratic functions when compared with conventional teaching approaches? Second, how do students perceive PBL with respect to their motivation and interest in learning? To respond to these questions, four objectives were established: (1) to determine participants' initial competency levels in solving quadratic function problems; (2) to develop and validate a PBL-based instructional sequence for teaching quadratic functions; (3) to evaluate the effectiveness of this instructional sequence in improving conceptual understanding, mathematical modelling, problem solving, mathematical communication, reasoning, and argumentation; and (4) to examine the relationship between students' motivation and interest in PBL and their resulting learning outcomes. The study hypothesises that PBL, through its collaborative, contextualised, and active learning environment, will generate statistically significant improvements across all assessed mathematical competencies, with particularly strong effects on higher-order competencies such as problem solving and reasoning. Nevertheless, differences in the magnitude of these effects are anticipated, thereby enabling the identification of competency areas in which PBL demonstrates the greatest educational value.

2. Materials and Methods

The study adopted a positivist (empirical-analytical or rationalist) paradigm and a quantitative approach to generate objective, measurable evidence and support the generalisation of findings (Herrera Castrillo, 2024; Vizcaíno Zúñiga et al., 2023). A quasi-experimental design was employed using two pre-existing first-year high school science classes, with Section A assigned as the control group receiving conventional instruction and Section B as the experimental group receiving PBL instruction (Ayari et al., 2025; Nguyen & Nguyen, 2025). The teaching methodology served as the independent variable, while pre-test and post-test assessments were conducted to establish baseline equivalence and evaluate the intervention's effects (Aminah et al., 2025). A descriptive-explanatory approach was also adopted to determine whether the observed outcomes could be attributed to the intervention rather than external factors (Vo & Nguyen, 2024).

The research was undertaken in a public secondary school located in Santo Domingo, Ecuador, serving an urban community characterised predominantly by a lower-middle socioeconomic background. The target population comprised first-year science students attending morning classes. Following institutional ethical approval and the acquisition of informed parental consent, a convenience sampling strategy was adopted, resulting in a total sample of 56 students, equally distributed between the control group (Section A, N=28) and the experimental group (Section B, N=28). Sample adequacy was established using the population proportion estimation formula at a 95% confidence level, assuming maximum variability ($p = q = 0.50$) and a 5% margin of error, producing a recommended sample size of approximately 57 participants (Vizcaíno Zúñiga et al., 2023). Eligibility criteria required participants to (a) be between 14 and 16 years of age, (b) be enrolled in the first year of the science stream at high school, (c) receive five mathematics lessons each week, and (d) have maintained continuous school attendance during previous academic years. Students requiring substantial curricular adaptations because of special educational needs were excluded from participation (Arias-Arroyo et al., 2017).

Three instruments were employed to collect the research data. The first was a researcher-developed achievement assessment administered as both a pre-test and post-test to evaluate mathematical competencies associated with quadratic functions (Arias-Gonzales, 2021). The instrument comprised 14 dichotomously scored items organised into five competency domains: Understanding of Key Concepts (4 items), Mathematical Modelling (2 items), Problem Solving (4 items), Mathematical Communication (2 items), and Reasoning and Argumentation (2 items), with each correct response receiving one point and each incorrect response receiving zero points (Yahya & Nurbayani, 2026). Raw scores were subsequently transformed to a 10-point grading scale. During pilot testing, the instrument was administered twice, with a one-week interval, to 15 students representing high-performing, low-performing, and control groups. Results from the Shapiro-Wilk test confirmed normal score distributions for both administrations ($p = 0.891$ and $p = 0.306$), while the test-retest analysis produced a Pearson correlation coefficient of $r = 0.696$ ($p = 0.004$), indicating satisfactory temporal reliability (Luzuriaga Jaramillo et al., 2023).

The second instrument consisted of a DSV checklist containing 16 items rated on a three-point Likert scale ranging from Non-Compliant (1) to Compliant (3). Ten experienced mathematics teachers evaluated the instructional sequence, and the resulting Cronbach's α coefficient of 0.791 demonstrated acceptable internal consistency (Rodríguez-Rodríguez & Reguant-Álvarez, 2020). The third instrument was the Spanish version of Part 1 of the TAC (v5.11), comprising 12 Likert-scale items scored from Strongly Disagree (1) to Strongly Agree (5). The questionnaire measured students' attitudes across several dimensions, including enjoyment, perceived usefulness of computers, intellectual challenge, and preferences regarding computer-supported versus conventional instruction. The instructional intervention itself was based on a teaching sequence which was specifically designed by the researchers and implemented over ten 45-minute lessons, equivalent to two weeks of classroom instruction (Rehman et al., 2024).

The instructional sequence was grounded in the constructivist perspectives of Piaget, Vygotsky, and Ausubel, incorporating the principles of cognitive conflict, the Zone of Proximal Development, and meaningful learning, respectively. Its overall design followed the didactic sequence framework. This framework culminated in the development of the five-stage IDRAC model. During the first stage, Introduction to the Problem (Lessons 1–2), students encountered authentic contextual problems, including scenarios involving fence optimisation, projectile motion, and small business profit maximisation. Working collaboratively, learners identified available information, determined missing knowledge, formulated preliminary hypotheses, and generated investigative questions, while the teacher acted solely as a facilitator without directly providing solutions. The second stage, Guided Discovery (Lessons 3–4), encouraged students to investigate their identified knowledge gaps through multiple learning resources, including concise explanatory texts, instructional videos, interactive GeoGebra activities, and structured worksheets. Throughout this stage, learners explored the principal characteristics of quadratic functions, including roots, discriminants, concavity, and vertices.

The third stage, Collaborative Resolution (Lessons 5–6), required student teams to integrate their newly acquired understanding by developing complete solutions, comparing alternative approaches, validating results, and constructing algebraic models, graphical representations, and contextual interpretations. Subsequently, students were presented with additional real-world applications drawn from physics, economics, biology, and sports, requiring them to identify quadratic relationships, formulate appropriate mathematical models, and solve contextual problems. Representative examples included determining the height of a water rocket using the function $h(t) = -5t^2 + 20t$ and analysing a revenue function represented by $I(x) = -2x^2 + 40x$.

The final stage, Closure, Reflection, and Evaluation (Lessons 9–10), involved group presentations, responses to questions posed by teachers and classmates, structured peer assessment, teacher evaluation, and self-assessment activities. Throughout the intervention, the control group received conventional teacher-centred instruction covering identical mathematical content over the same instructional duration. All research procedures complied fully with recognised ethical principles, including respect for persons, beneficence, non-maleficence, and justice, following formal institutional approval before the commencement of data collection. Written informed consent was obtained from the legal guardians of all 56 participants, while student assent was also secured before participation commenced. Participant confidentiality was protected by assigning numerical identification codes to all datasets, with the code key stored separately from the research data on a secure, restricted-access device (Torres Paez & Londoño, 2024). The TAC (v5.11) instrument was used in accordance with its open-access availability for educational research purposes (Zahid & Nawab, 2025).

Statistical analyses were conducted using SPSS version 25. Descriptive statistics included the calculation of means, medians, standard deviations, percentile distributions, and score ranges for both groups at the pre-test and post-test stages. Data normality was evaluated using the Shapiro-Wilk test (Luzuriaga Jaramillo et al., 2023). Because the assumption of normality was not satisfied for comparisons between groups, differences in median scores were analysed using the non-parametric Mann-Whitney U test (Bautista-Díaz et al., 2020). Statistical significance for all analyses was established at $p < 0.05$ (Vélez Córdova & Arteaga Pita, 2022).

3. Results and Discussion

Initial analyses were conducted using the pre-test results to establish baseline performance and verify the equivalence of the two groups before the intervention. In the control group, 14.29% of the students achieved a satisfactory level of performance (AAR: 7.00–8.99), whereas the remaining 85.71% obtained scores below the satisfactory threshold (Score < 7). Similarly, within the experimental group, 25% of the participants reached the AAR category, while 75% failed to attain the minimum satisfactory score. Table 1 presents the qualitative and quantitative grading scale adopted for classifying student performance throughout the study.

Table 1: Qualitative–Quantitative Scale for Assessment of Learning Outcomes.

Qualitative Scale	Abbrev.	Quantitative Scale	Interpretation
Mastery of Required Learning Outcomes	DAR	9.00 – 10.00	Excellent
Achieves Required Learning Outcomes	AAR	7.00 – 8.99	Satisfactory
Close to Achieving Required Learning Outcomes	PAAR	4.01 – 6.99	Approaching
Does not Achieve Required Learning Outcomes	NAAR	≤ 4.00	Insufficient

Note: The minimum passing score for course approval is 7.00.

Table 2: Descriptive Statistics for Pre-Test Scores by Group.

Statistic	Control Group (N=28)	Experimental Group (N=28)	Difference
Mean	4.90	5.36	+0.46
Median	4.64	5.71	+1.07
Mode	3.21	7.50	+4.29
Standard Deviation	1.90	2.25	+0.35
Minimum	1.43	1.43	0.00
Maximum	7.86	8.93	+1.07
75th Percentile	6.25	6.96	+0.71

Note: Scores reported on the 0–10 scale established by Ecuador's Ministry of Education. Mann-Whitney $U = 320.50$, $p = 0.238$.

Descriptive analysis indicated slightly higher baseline scores for the experimental group than for the control group. The experimental group achieved a mean score of 5.36 compared with 4.90 for the control group, while the corresponding median scores were 5.71 and 4.64, representing differences of 0.46 and 1.07 points, respectively. However, the Shapiro-Wilk normality test revealed that neither dataset followed a normal distribution (control: $p = 0.016$; experimental: $p = 0.044$). Consequently, non-parametric statistical procedures were employed for group comparisons. The Mann-Whitney U test produced a value of $U = 320.50$ ($z = -1.18$, $p = 0.238$), demonstrating that no statistically significant differences existed between the two groups prior to the intervention. The descriptive statistics for the pre-test are presented in Table 2.

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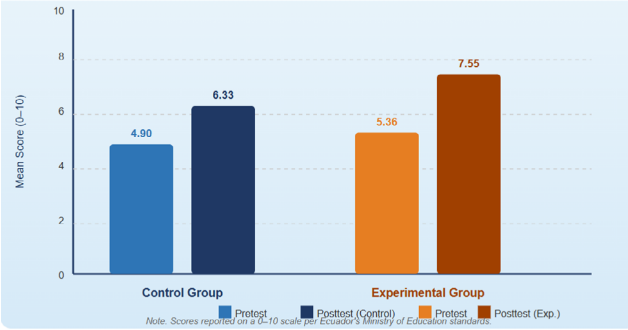
Figure 1: Pre-Test and Post-Test Mean Scores by Group.



Note: Both groups started from comparable baselines; the experimental group demonstrated greater gains after the PBL intervention.

Following the intervention, the proportion of students in the experimental group achieving or surpassing the minimum passing standard (AAR or DAR) increased from the pre-test to the post-test, rising by 14.29 percentage points. The post-test mean score of the experimental group (M = 7.55) exceeded that of the control group (M = 6.33) by 1.22 points. Assessment of data normality using the Shapiro-Wilk test indicated that the post-test scores for both groups remained non-normally distributed (control: $p = 0.009$; experimental: $p < 0.001$). Consequently, the Mann-Whitney U test was applied, yielding $U = 267.00$, $z = -2.36$, and $p = 0.018$. These results demonstrate a statistically significant difference between the experimental and control groups following the implementation of the PBL intervention (Reid O'Connor & Norton, 2022). This outcome is consistent with previous research reporting superior mathematical learning outcomes among students taught through PBL compared with those receiving conventional direct instruction (Buri Sánchez, 2024; Laine & Mahmud, 2022).

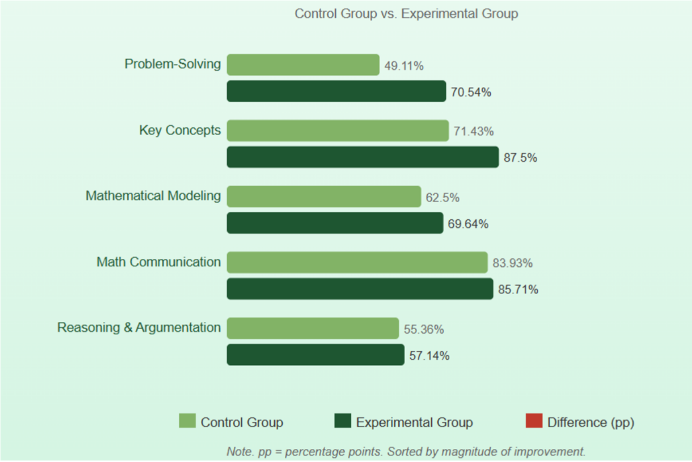
Analysis at the competency level revealed a differentiated pattern of improvement, representing one of the study's most significant findings. The greatest enhancement was observed in problem-solving competency, which increased by 21.43 percentage points, rising from 49.11% to 70.54%. Understanding of key concepts also showed substantial improvement, increasing by 16.07 percentage points from 71.43% to 87.50%. Mathematical modelling demonstrated a more modest gain of 7.14 percentage points (Haron et al., 2021). In contrast, mathematical communication and reasoning and argumentation exhibited only marginal improvements, with each competency increasing by 1.78 percentage points. A summary of these competency-specific outcomes is presented in Table 3.

Table 3: Post-Test Correct Response Rates by Mathematical Competency and Group.

Mathematical Competency	Control Freq.	Control %	Exp. Freq.	Exp. %	Difference
Understanding Key Concepts (4 Items)	80 / 112	71.43%	98 / 112	87.50%	+16.07 pp
Mathematical Modelling (2 Items)	35 / 56	62.50%	39 / 56	69.64%	+7.14 pp
Problem-Solving (4 Items)	55 / 112	49.11%	79 / 112	70.54%	+21.43 pp
Mathematical Communication (2 Items)	47 / 56	83.93%	48 / 56	85.71%	+1.78 pp
Reasoning & Argumentation (2 Items)	31 / 56	55.36%	32 / 56	57.14%	+1.78 pp

Note: Frequency = Total correct responses across all students per dimension. Maximum possible = n × items (e.g., 28 × 4 = 112 for 4-item dimensions). pp = Percentage Points.

Figure 2: Post-Test Scores by Mathematical Competency (%).

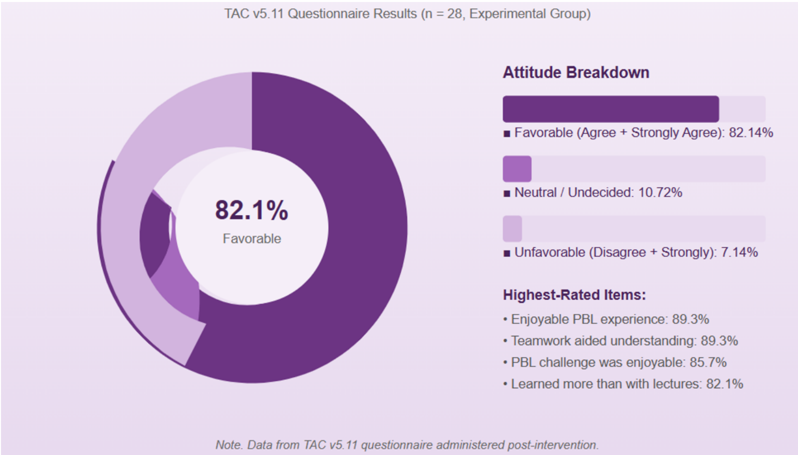


The observed pattern is both theoretically meaningful and consistent with the intended design of the instructional intervention. Because PBL is fundamentally centred on authentic problem solving, the greatest improvement was expected to occur in competencies directly associated with this process across all five stages of the IDRAC sequence. The findings support this expectation and align with Meneses Espinal and Peñaloza Gelvez (2019), who argued that problem solving cultivates perseverance, curiosity, and disciplined thinking. Likewise, the notable improvement in conceptual understanding is compatible with Ausubel's theory of meaningful learning, which suggests that students construct more durable conceptual knowledge when they engage with essential mathematical ideas, such as the vertex, concavity, and discriminant, while solving authentic problems rather than relying on memorisation alone (Torres Paez & Londoño, 2024).

Conversely, the relatively modest gains observed in mathematical communication and reasoning indicate that exposing students to problem-solving activities alone is insufficient for developing these competencies. Instead, these skills appear to require deliberate instructional scaffolding and targeted learning opportunities. Although previous research has demonstrated that PBL can substantially strengthen mathematical communication, Salama et al. (2025) incorporated dedicated communication sessions and assessment rubrics specifically designed for that purpose. In contrast, the present study assumed that communication skills would emerge naturally through collaborative problem-solving activities rather than through explicit instruction. However, effective evaluation within PBL should extend beyond the quality of final solutions to include the collaborative process itself, encompassing teamwork dynamics, task allocation, and the quality of mathematical reasoning demonstrated throughout the learning process. Similarly, the moderate improvement observed in mathematical modelling (+7.14 percentage points) can be attributed to the cognitive demands involved in converting authentic contextual situations into quadratic mathematical representations, a process requiring students to transform informal reasoning into formal mathematical models (Ávila Moreno, 2019).

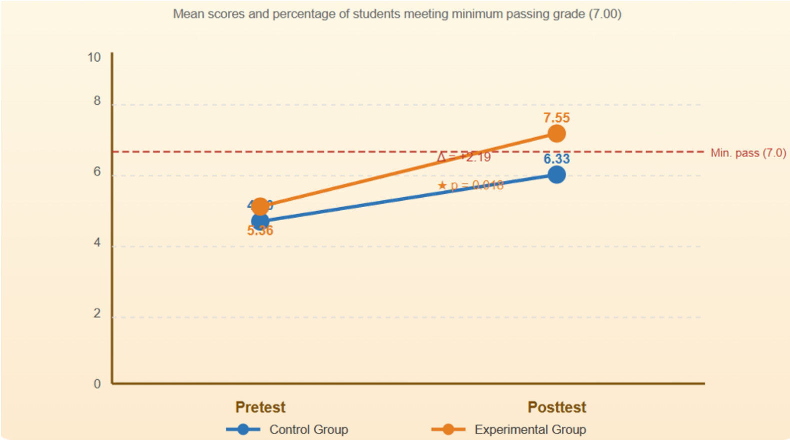
The instructional sequence was further supported through expert validation conducted by ten experienced mathematics teachers. Evaluation of the checklist produced a Cronbach's α coefficient of 0.791, confirming the overall reliability and suitability of the IDRAC sequence (Rodríguez-Rodríguez & Reguant-Álvarez, 2020). Although none of the experts identified any indicator as completely non-compliant, several areas requiring refinement were highlighted. Specifically, 30% of the reviewers considered the authentic problem component only partially satisfactory, suggesting that the farmer's fence optimisation scenario may have lacked relevance for students from an urban environment. Furthermore, 70% of the experts judged the self-assessment and peer-assessment component to be only partially adequate, noting that the reflection activities did not sufficiently promote metacognitive thinking. These observations are consistent with the recommendation of Lojano Guerrero (2023) which advocates comprehensive assessment practices that integrate cooperative evaluation, fulfilment of learning expectations, equitable task distribution, and structured peer evaluation as essential components of effective PBL implementation (Barraza Macías et al., 2020)2020.

Figure 3: Student Attitude Distribution Toward PBL Methodology.



Accordingly, future applications of the instructional sequence should allocate dedicated class time for structured metacognitive reflection supported by carefully designed assessment rubrics. Students' perceptions of PBL were generally favourable, with 82.14% expressing positive overall attitudes by selecting either "agree" or "strongly agree." The highest levels of agreement were associated with enjoyment of the learning experience (89.3%) and the value of teamwork in facilitating understanding, findings that correspond with the affective benefits of PBL reported by Silitonga et al. (2025) and Luy-Montejo (2019). Nevertheless, 46.5% of the participants indicated that they experienced uncertainty when expected to solve problems without explicit teacher guidance. This response mirrors the observations of prior studies, that found that learners accustomed to teacher-centred instruction frequently experience difficulty when transitioning to inquiry-based learning environments. These findings reinforce the importance of implementing PBL progressively, supported by explicit modelling of inquiry processes and stronger instructional scaffolding before students are expected to engage independently in self-directed problem solving.

Figure 4: Learning Progression: Pre-Test to Post-Test Score Trends.



Note: Learning progression from pre-test to post-test for both groups. The orange line (experimental) crosses the minimum passing grade threshold (7.00) while the blue line (control) remains below it.

Figure 1 illustrates the mean pre-test and post-test scores for both the control and experimental groups. At baseline, the two groups demonstrated comparable performance, whereas following the intervention the control group achieved a mean post-test score of 6.33, while the experimental group attained a higher mean score of 7.55 on the 0–10 grading scale. Figure 2 presents a horizontal grouped bar chart comparing post-test performance across the five mathematical competencies. The largest improvement was observed in problem solving, which increased by 21.43 percentage points, followed by understanding of key concepts with a gain of 16.07 percentage points. By comparison, mathematical communication and reasoning and argumentation exhibited only marginal improvements of 1.78 percentage points, highlighting the differential impact of PBL across competency domains.

Students' perceptions of PBL are summarised in Figure 3 through the results of the TAC questionnaire. Overall, 82.14% of participants expressed favourable attitudes towards the instructional approach, 10.72% reported neutral perceptions, and 7.14% indicated unfavourable opinions. Among those with positive perceptions, 89.3% agreed that the learning experience was enjoyable and that collaborative teamwork enhanced their understanding of the subject matter. Figure 4 depicts the learning progression of both groups from the pre-test to the post-test using trend lines together with the minimum passing benchmark. The figure demonstrates clear improvement in both groups; however, the experimental group exhibited substantially greater progress, consistent with the statistically significant difference identified in the post-test analysis ($p = 0.018$). Figure 5 summarises the expert evaluation of the IDRAC sequence by presenting the proportion of reviewers who judged each of the seven instructional indicators as fully compliant. High levels of agreement were observed across most dimensions, with Guided Discovery receiving the highest level of compliance (90%). In contrast, the self-assessment and peer-assessment component obtained the lowest level of full compliance, with only 30% of experts rating this indicator as fully satisfactory, identifying it as the principal area requiring refinement in future implementations.

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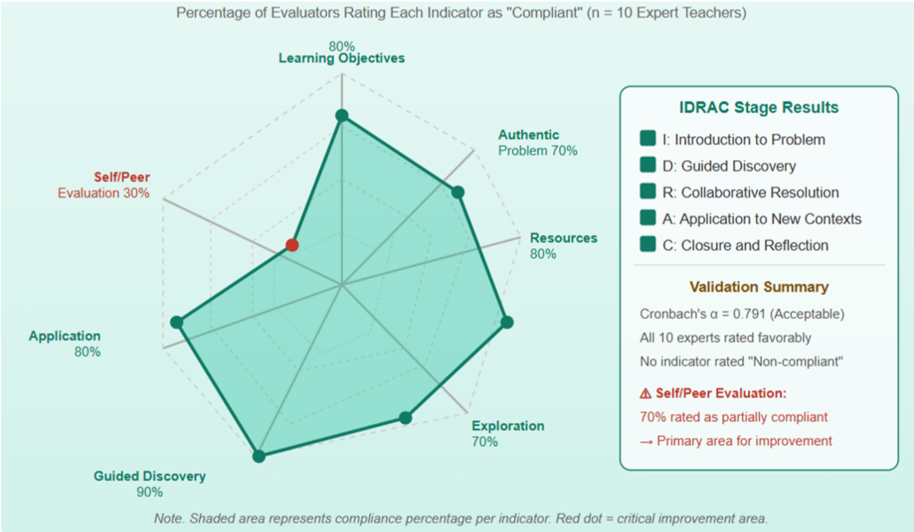
Figure 5: IDRAC Didactic Sequence Model—Validation Expert Ratings.



Note: Radar chart displaying expert validation scores for each IDRAC stage indicator. The red dot marks the critical improvement area: self-assessment and peer-assessment (30% full compliance).

These findings have important implications within the broader context of educational reform in Ecuador. The results of the present study provide empirical support for these policy initiatives while also identifying several practical considerations for their successful implementation. Specifically, teacher preparation programmes, both pre-service and in-service, should place greater emphasis on designing authentic mathematical problems, providing explicit scaffolding for mathematical communication and reasoning, and incorporating structured self-assessment and peer-assessment practices into classroom instruction. As highlighted by prior literature, mathematical literacy is closely associated with students' academic achievement. Therefore, equipping learners with opportunities to engage collaboratively in authentic problem-solving activities is essential for strengthening mathematical literacy and supporting their broader academic development.

4. Limitations and Scope for Future Research

The findings of this study should be interpreted in light of several limitations. First, the quasi-experimental design and the use of convenience sampling restrict the generalisability of the results. Consequently, the findings may not be directly applicable to other schools, classrooms, or student populations, and their external validity is confined to the context of an urban public school in Santo Domingo, Ecuador. Second, the intervention was implemented over only ten 45-minute lessons spanning two academic weeks, making it difficult to determine whether the observed competency gains would be maintained over time or transferred to other areas of mathematics beyond quadratic functions. Third, the achievement instrument consisted exclusively of dichotomously scored items (correct/incorrect), limiting its ability to capture the depth of students' mathematical reasoning and the quality of their communication, both of which are central to higher-order mathematical competencies. Finally, because one of the researchers also served as the instructor for the experimental group, the possibility of experimenter bias cannot be entirely dismissed, although this risk was reduced through the use of standardised procedures and objective assessment instruments. These limitations provide important directions for future research.

Future research should examine the long-term effects of PBL and its transferability to other mathematical topics, such as trigonometry, statistics, and calculus. Replication across different content areas would determine whether the observed pattern of stronger gains in problem solving and conceptual understanding than in communication and reasoning is consistent across mathematics. Incorporating qualitative methods, including classroom observations, interviews, and interaction analyses, would provide deeper insights into how PBL develops individual competencies. Further studies should also investigate the integration of digital technologies, such as GeoGebra and AI-supported platforms, while considering institutional and teacher-related factors that may influence implementation. Finally, larger and more diverse mixed-methods studies are recommended to enhance the generalisability of the findings and provide stronger evidence for educational policy and practice.

5. Conclusion

This study evaluated the effectiveness of PBL in developing 21st century mathematical competencies during the teaching of quadratic functions. The findings indicate that PBL significantly improved students' academic performance, with the experimental group achieving a post-test mean score 1.22 points higher than the control group (Mann-Whitney U test, $p = 0.018$), supporting previous evidence of its effectiveness in the Ecuadorian context. The greatest gains were observed in problem solving (+21.43 percentage points) and understanding of key concepts (+16.07 percentage points), whereas mathematical communication and reasoning and argumentation showed only modest improvements (+1.78 percentage points), highlighting the need for explicit scaffolding and targeted assessment. Expert evaluation also confirmed the suitability of the IDRAC sequence for implementing PBL (Cronbach's $\alpha = 0.791$), while identifying opportunities to improve authentic problem design and strengthen structured self-assessment and peer-assessment. Students responded positively to PBL, with 82.14% expressing favourable attitudes, although many required greater teacher guidance, emphasising the importance of gradual implementation. Overall, the findings demonstrate that PBL is an effective approach for strengthening mathematical competencies and reducing the gap between school mathematics and real-world demands, provided it is supported by authentic problem design, competency-specific scaffolding, appropriate assessment, and ongoing teacher professional development.

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